

## Lecture Note on Stock Index Futures Pricing.

Long before trading in stock index futures began, investors were following general stock behavior by using weighted indexes. The price –equally weighted index is constructed by adding up the prices of the stocks in the index and then dividing by a divisor:

Index ==  $[1/\text{Divisor}] [ P_1 + \dots + P_N ]$  Where  $P_i$  is the price of stock  $i$  and  $N$  is the number of stocks comprising the index. Both the Dow Jones and the Major Market Index (MMI) the underlying index of the Chicago Board of Trade's stock index futures contract, are price equally weighted indices.

1. Suppose that on a given day we observe the following prices:

Stock	Price
IBM	\$120
Unisys	\$ 25
Xerox	\$ 60

A price-equally weighted index of these stocks, the High Tech Three has a value of 10.25.

a. What is the current value of the divisor?

$$10.25 = [1/\text{divisor}] \{ \$120 + \$25 + \$60 \} \implies 10.25 = 1/\text{divisor} \{ \$205 \}$$

$$10.25/205 = 1/\text{divisor}$$

$$.05 = 1/\text{divisor} \implies \text{Divisor} = 1/.05 = 20$$

b. Suppose each point of the index represents \$500. How many shares of each stock must we hold to create a portfolio that mimics the index?

$$\text{The total value of the index would be: } \$500 \times 10.25 = \$5,125$$

$$\{ \text{Number of Shares} \} (\$120 + \$60 + \$25) = \$5,125$$

$$\text{Number of Shares} = \$5,125 / \$205 = 25 \text{ shares}$$

c. Suppose IBM undergoes a three for one stock split and there are no other price changes. What is the new value of the divisor?

If IBM undergoes a 3 for 1 stock split, then its new value is  $\$120/3$  or  $\$40$ . Thus the new sum of stock prices is  $\$40 + \$60 + \$25$  or  $\$125$ . The new divisor is set so that the value of the index stays the same:

$$10.25 = [1/\text{Divisor}] \{\$125\} \Rightarrow 125/10.25 \text{ or } 12.1951$$

d. How many shares of each stock must we now hold to create a portfolio that mimics the index? What is the next rebalancing cost?

The total value of the index is still  $\$5,125$ . Thus,

The Number of Shares  $[\$40 + \$60 + \$25] = \$5,125$

$$\# \text{ of Shares} = 5,125/125 = 41$$

The value of both mimicking portfolios is the same, therefore, the rebalancing requires no net expenses except any transactions costs that may be associated with the IBM stock split.